PHYSICS ACADEMY
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JEST

(MATHEMATICAL PHYSICS)

PREVIOUS YEAR'S QUESTIONS WITH ANSWER (CHAPTER-WISE)

- ☞ MATRIX ALGEBRA
- ☞ VECTOR ANALYSIS
- ☞ FOURIER SERIES, FOURIER & LAPLACE TRANSFROMATION
- ☞ COMPLEX ANALYSIS
- ☞ DIFFERENTIAL EQUATION
- ☞ OTHER QUESTIONS

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14, RAIPUR ROAD, NEAR SURVEY CHOWK, DEHRADUN CONTACT:- +919045460409, +919870827730 website:- www. www.careerspectra.in

MATRICS ALGEBRA

1. For an $N \times N$ matrix consisting of all ones, **[JEST-2012]**

(a) All eigenvalues = 1 (b) All eigenvalues = 0

(c) The eigenvalues are 1,2,....N

(d) One eigenvalue = N , the others = 0

2. Given a matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, which of the following represents $\cos\left(\frac{\pi M}{6}\right)$. **[JEST-2016]**

(a) $\frac{1}{2}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ (c) $\frac{\sqrt{3}}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

3. Let $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ and $M = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. Similarity, transformation of M to Λ can be performed by. **[JEST 2017]**

(a) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$

(c) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ -3i & 11 \end{pmatrix}$ (d) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & 3i \\ -3i & 1 \end{pmatrix}$

4. If $\rho = \frac{\left[I + \frac{1}{\sqrt{3}}(\sigma_x + \sigma_y + \sigma_z)\right]}{2}$, where σ's are the Pauli matrices and I is the identity matrix, then the trace of ρ^{2017} is. **[JEST 2017]**

(a) 2^{2017} (b) 2^{-2017} (c) 1 (d) $\frac{1}{2}$

5. Two of the eigenvalues of the matrix $A = \begin{pmatrix} a & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are 1 and -1. What is the third eigenvalue? **[JEST-2018]**

(a) 2 (b) 5 (c) -2 (d) -5

6. Consider two $n \times n$ matrices, A and B such that $A + B$ is invertible. Define two Matrices, $C = A(A+B)^{-1}B$ and $D = B(A+B)^{-1}A$. Which of the following relations always hold true? **[JEST-2019]**

(a) C= D (b) C^{-1}= D

(c) BCA = ADB (d) C ≠ D

7. Let A be a hermitian matrix, and C and D be the unitary matrices. Which one of the following matrices is unitary? **[JEST-2019]**

(a) $C^{-1}AC$ (b) C^{-1} DC (c) $C^{-1}AD$ (d) A^{-1} CD

8. Consider a 2x2 matrix $A = \begin{pmatrix} 1 & 13 \\ 0 & 1 \end{pmatrix}$ what is A^{27}? **[JEST-2019]**

(a) $\begin{pmatrix} 1 & 13 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 13^{27} \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 27 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 351 \\ 0 & 1 \end{pmatrix}$

VECTOR ANALYSIS

9. If the distribution function of x is $f(x) = xe^{-x/\lambda}$ over the interval $0 < x < \infty$, the mean value of x is. **[JEST-2013]**

(a) λ (b) 2λ (c) $\frac{\lambda}{2}$ (d) 0

10. What is the equation of the plane which is tangent to the surface $xyz = 4$ at the point (1,2,2)? **[JEST-2017]**

(a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$

(c) $x + 4y + z = 0$ (d) $2x + y + z = 6$

11. Let $\vec{r}$ be the position vector of a point on a closed contour C . What is the value of the line integral $\oint \vec{r}.d\vec{r}$? **[JEST-2019]**

(a) 0 (b) ½ (c) 1 (d) π

12. Which one of the following vectors lie along the line of intersection of the two planes x+3y-z=5 and 2x-2y+4z=3? **[JEST-2019]**

(a) $10\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (b) $10\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$

(c) $10\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ (d) $10\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

13. What is the angle (in degrees) between the surfaces $y^2 + z^2 = 2$ and $y^2 - x^2 + = 0$ at the Point (1, -1,1). **[JEST-2019]**

FOURIER SERIES, FOURIER & LAPLACE TRANSFROMATION

14. The Laplace transformation of e^{-2t} sin 4t is: **[JEST-2014]**

(a) $\frac{4}{s^2+4s+25}$ (b) $\frac{4}{s^2+4s+20}$ (c) $\frac{4s}{s^2+4s+20}$ (d) $\frac{4s}{2s^2+4s+20}$

15. The Dirac delta function $\delta(x)$ satisfies the relation $\int_{-\infty}^{\infty} f(x)\ \delta(x)dx = f(0)$ for a well behaved function $f(x)$. If x has the dimension of momentum then **[JEST-2014]**

(a) $\delta(x)$ has the dimension of momentum

(b) $\delta(x)$ has the dimension of $(\text{momentum})^2$

(c) $\delta(x)$ is dimensionless

(d) $\delta(x)$ has the dimension of $(\text{momentum})^{-1}$

16. $\int_{-\infty}^{+\infty}(x^2+1)\delta)(x^2-3x+2)dx$ =? **[JEST 2017]**

(a) 1 (b) 2 (c) 5 (d) 7

17. The Fourier transform of the function $\frac{1}{x^4+3x^2+2}$ up to proportionality constant is: **[JEST 2017]**

(a) $\sqrt{2}\exp(-k^2)-\exp(-2k^2)$
(b) $\sqrt{2}\exp(-|k|)-\exp\left(-\sqrt{2}|k|\right)$
(c) $\sqrt{2}\exp\left(-\sqrt{|k|}\right)-\exp\left(-\sqrt{2|k|}\right)$
(d) $\sqrt{2}\exp\left(-\sqrt{2}k^2\right)-\exp(-2k^2)$

18. The function $f(x)=\cos hx$ which exists in the range $-\pi\le x\le\pi$ is periodically repeated between $x=(2m-1)\pi$ and $(2m+1)\pi$, where $m=-\infty\ to\ \infty$.

Fourier series, indicate the correct relation at $x = 0$. **[JEST 2017]**

(a) $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1-n^2}=\frac{1}{2}\left(\frac{\pi}{\cosh\pi}-1\right)$ (b) $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1-n^2}=2\frac{\pi}{\cosh\pi}$
(c) $\sum_{n=-\infty}^{\infty}\frac{(-1)^{-n}}{1+n^2}=2\frac{\pi}{\sinh\pi}$ (d) $\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1+n^2}=\frac{1}{2}\left(\frac{\pi}{\sinh\pi}-1\right)$

19. The Laplace transform of $\frac{(\sin(at)-at\cos(at))}{(2a^3)}$ is. **[JEST-2018]**

(a) $\frac{2as}{(s^2+a^2)^2}$ (b) $\frac{s^2-a^2}{(s^2+a^2)^2}$ (c) $\frac{1}{(s+a)^2}$ (d) $\frac{1}{(s^2+a^2)^2}$

20. $\pi\int_{-\infty}^{\infty}\exp(-|x|)\delta\ (\sin(\pi x))dx$, where $\delta(\ldots)$is Dirac distribution, is **[JEST-2018]**

(a) 1 (b) $\frac{e+1}{e-1}$ (c) $\frac{e-1}{e+1}$ (d) $\frac{e}{e+1}$

21. What is the value of the integral $\int_{-\infty}^{+\infty}dx\delta(x^2-\pi^2)$ cos x? **[JEST-2019]**

(a) π (b) $-\frac{1}{2\pi}$ (c) $-\frac{1}{\pi}$ (d) 0

COMPLEX ANALYSIS

22. The value of the integral $\int_0^{\infty}\frac{\ln x}{(x^2+1)^2}$ dx is. **[JEST-2012]**

(a) 0 (b) $\frac{-\pi}{4}$ (c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

23. Compute $\log_{z\to0}\frac{Re(z^2)+Im\ (z^2)}{z^2}$. **[JEST-2013]**

(a) The limit does not exist (b) 1
(c) -*i* (d) -1

24. The value of limit $\lim_{z\to i}\frac{z^{10}+1}{z^6+1}$ is equal to. **[JEST-2014]**

(a) 1 (b) 0 (c) $\frac{-10}{3}$ (d) $\frac{5}{3}$

25. The value of integral $I = \oint_c \frac{\sin z}{2z-\pi}dz$ with c a circle $|z| = 2$, is **[JEST-2014]**

(a) 0 (b) $2\pi i$ (c) πi (d) πi

26. Given an analytic function $f(z) = \phi(x,y) + i\psi(x,y)$, where $\phi(x,y) = x^2 + 4x - y^2 + 2y$. If C is a constant, which of the following relations is true? **[JEST-2015]**

(a) $\psi(x,y) = x^2y + 4y + C$ (b) $\psi(x,y) = 2xy - 2x + C$
(a) $\psi(x,y) = 2xy + 4y - 2x + C$ (a) $\psi(x,y) = x^2y - 2x + C$

27. Which one is the image of the complex domain $\{z|xy \geq 1, x + y > 0\}$ under the mapping $f(z) = z^2$, if z = x+iy? **[JEST 2017]**

(a) $\{z|xy \geq 1, x + y > 0\}$ (b) $\{z|x \geq 2, x + y > 0\}$
(c) $\{z|y \geq 2\forall x\}$ (d) $\{z|y \geq 1\forall x\}$

28. The integral $\int_{-\infty}^{\infty}\frac{\cos x}{x^2+1}dx$ is. **[JEST-2018]**

(a) $\frac{\pi}{e}$ (b) πe^{-2} (c) π (d) zero

29. Consider the function $f(x,y) = |x| - i|y|$. In which domain of the complex plane is this function analytic? **[JEST-2019]**

(a) First and second quadrants (b) Second and third quadrants
(c) Second and fourth quadrants (d) Nowhere

DIFFERENTIAL EQUATION

30. What are the solutions of $f"(x) - 2f'(x) + f(x) = 0$? **[JEST-2014]**

(a) c_1e^x/x (b) $c_1x + c_2/x$
(c) $c_1xe^x + c_2$ (d) $c_1e^x + c_2xe^x$

31. What is the maximum number of extrema of the function $f(x) = P_k(x)e^{-\left(\frac{x^4}{4}+\frac{x^2}{2}\right)}$, where $x \epsilon (-\infty, \infty)$ and $P_k(x)$ is an arbitrary polynomial of degree k? **[JEST-2015]**

(a) $k + 2$ (b) $k + 6$ (c) $k + 3$ (d) k

32. Consider the differential equation $G'(x) + kG(x) = \delta(x)$; where k is a constant. Which of the following statements is true? **[JEST-2015]**

(a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$.

(b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not.

(c) $G(x)$ is discontinuous at $x = 0$.

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depends on the value of k.

OTHER QUESTION

33. If [x] denotes the greatest integer not exceeding x, then $\int_0^\infty [x]e^{-x}dx$. **[JEST-2012]**

(a) $\frac{1}{e-1}$ (b) 1 (c) $\frac{e-1}{e}$ (d) $\frac{1}{e^2-1}$

34. As $x \to 1$, the infinite series $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots$ **[JEST-2012]**

(a) Diverges (b) Converges to unity

(c) Converges to $\frac{\pi}{4}$ (d) None of the above

35. What is the value of the following series? $\left(1 + \frac{1}{2!} + \frac{1}{4!} + \cdots\right)^2 - \left(1 + \frac{1}{3!} + \frac{1}{5!} + \cdots\right)^2$. **[JEST-2012]**

(a) 0 (b) e (c) e^2 (d) 1

36. An unbiased die is cast twice. The probability that the positive difference (bigger - smaller) between the two numbers is 2 is. **[JEST-2012]**

(a) 1/9 (b) 2/9 (c) 1/6 (d) 1/3

37. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double headed one? **[JEST-2013]**

(a) 0.99 (b) 0.925 (c) 0.75 (d) 0.01

38. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to. **[JEST-2013]**

(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

39. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x axis. The probability that they meet after n steps is. **[JEST-2013]**

(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

40. What is the value of the following series? $\left(1-\frac{1}{2!}+\frac{1}{4!}\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}\right)^2$. **[JEST-2013]**

(a) 0 (b) e (c) e^2 (d) 1

41. If the distribution function of x is $f(x)=xe^{-x/\lambda}$ over the interval $0 < x < \infty$, the mean value of x is. **[JEST-2013]**

(a) λ (b) 2λ (c) $\frac{\lambda}{2}$ (d) 0

42. The value of $\int_{0.2}^{2.2} xe^x\,dx$ by using the one-segment trapezoidal rule is closed to. **[JEST-2014]**

(a) 11.672 (b) 11.807 (c) 20.099 (d) 24.119

43. If two ideal dice are rolled once, what is the probability of getting at least one '6'? **[JEST-2015]**

(a) 11/36 (b) 1/36 (c) 10/36 (d) 5/36

44. The Bernoulli polynominals $B_n(s)$ are defined by, $\frac{xe^{xs}}{e^x-1}=\sum B_n(s)\frac{x^n}{n!}$. Which one of the following relations is true? **[JEST-2015]**

(a) $\frac{xe^{x(1-s)}}{e^x-1}=\sum B_n(s)\frac{x^n}{(n+1)!}$ (b) $\frac{xe^{x(1-s)}}{e^x-1}=\sum B_n(s)(-1)^n\frac{x^n}{(n+1)!}$

(c) $\frac{xe^{x(1-s)}}{e^x-1}=\sum B_n(-s)(-1)^n\frac{x^n}{n!}$ (d) $\frac{xe^{x(1-s)}}{e^x-1}=\sum B_n(s)(-1)^n\frac{x^n}{n!}$

45. The sum $\sum_{m=1}^{99}\frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to. **[JEST-2015]**

(a) 9 (b) $\sqrt{99}-1$ (c) $\frac{1}{(\sqrt{99}-1)}$ (d) 11

46. The mean value of random variable x with probability density $p(x)=\frac{1}{\sigma\sqrt{2\pi}}.\exp\left[-\frac{(x^2+\mu x)}{(2\sigma^2)}\right]$ is: **[JEST-2016]**

(a) 0 (b) $\frac{\mu}{2}$ (c) $\frac{-\mu}{2}$ (d) σ

47. The sum of the infinite series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is. **[JEST-2016]**

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

48. A semicircular piece of paper is folded to make a cone with the centre of the semicircle as the apex. The half-angle of the resulting cone would be: **[JEST-2016]**

(a) 90^0 (b) 60^0 (c) 45^0 (d) 30^0

49. Suppose that we toss two fair coins hundred times each. The probability that the same number of heads occur for both coins at the end of the experiment is:

(a) $\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}$ (b) $2\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}^2$

(c) $\frac{1}{2}\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}^2$ (d) $\left(\frac{1}{4}\right)^{100}\sum_{n=0}^{100}\binom{100}{n}^2$

50. The integral $I = \int_1^{\infty}\frac{\sqrt{x-1}}{(1+x)^2}dx$ is. **[JEST 2017]**

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2\sqrt{2}}$ (c) $\frac{\sqrt{\pi}}{2}$ (d) $\sqrt{\frac{\pi}{2}}$

51. An electronic circuit with 10000 components performs its intended function success fully with a probability 0.99 if there are no faulty components in the circuit. The probability that there are faulty components is 0.05 . if there are faulty components, the circuit perform successfully with a probability 0.3 . The probability that the circuit performs successfully is $\frac{x}{10000}$. What is *x* ? . **[JEST 2018]**

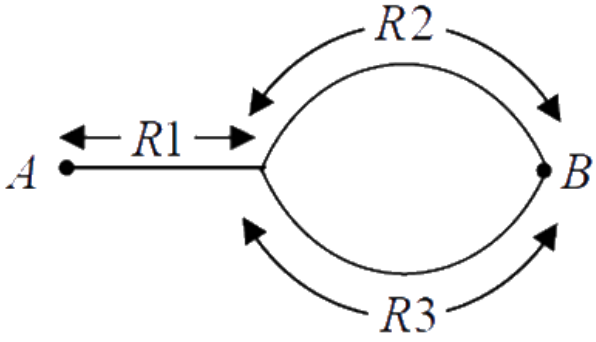
52. If an abelian group is constructed with two distinct elements *a* and *b* such that $a^2 = b^2 = I$, where *I* is the group identity. What is the order order of the smallest abelian group containing *a*,*b* and *I* ? **[JEST 2018]**

53. If $F(x,y) = x^2 + y^2 + xy$, its Legendre transformed function G(u,v), upto a multiplicative constant, is. **[JEST 2018]**

(a) $u^2 + v^2 + uv$ (b) $u^2 + v^2 - uv$

(c) $u^2 + v^2$ (d) $(u+v)^2$

54. A person plans to go from town *A* to town *B* by taking either the route (*R*1+*R*2) with Probability (1/2) or the route (R1+R3) with probability (1/2) (see figure). Further, there is a probability (1/3) that *R*1 is blocked, a (1/3) that *R*2 is blocked, and a probability (1/3) that *R*3 is blocked. What is the probability that he/she would reach town *B* ? **[JEST 2019]**



(a) $\frac{8}{9}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

55. Consider a function $f(x) = P_k(x)e^{-(x^4+2x^2)}$ in the domain $x \in (-\infty, \infty)$, where P_k is any polynomial of degree k . What is the maximum possible number of extrema of the function? **[JEST 2019]**

(a) k + 3 (b) k - 3 (c) k + 2 (d) k +1

56. The Euler polynomials are defined by $\frac{2e^{xs}}{e^x+1} = \sum_{n=0}^{\infty} E_n(s)\frac{x^n}{n!}$ What is the value of $E_5(2) + E_5(3)$? **[JEST 2019]**

ANSWER KEY

1.	D	2.	B	3.	A	4.	C	5.	B	6.	A
7.	B	8.	D	9.	B	10.	D	11.	A	12.	B
13.	60	14.	B	15.	D	16.	D	17.	B	18.	D
19.	D	20.	B	21.	C	22.	B	23.	A	24.	D
25.	C	26.	C	27.	*	28.	A	29.	C	30.	D
31.	C	32.	C	33.	A	34.	C	35.	D	36.	B
37.	C	38.	D	39.	A	40.	D	41.	B	42.	C
43.	A	44.	D	45.	A	46.	A	47.	D	48.	D
49.	D	50.	B	51.	9555	52.	4	53.	B	54.	C
55.	A	56.	64								